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SIMILARITY BASED FUZZY INFERIOR RATIO FOR SOLVING MULTICRITERIA DECISION MAKING PROBLEMS

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ABSTRACT

The method of Fuzzy Inferior Ratio (FIR) has been recognized as one of advantageous methods in multi criteria decision-making under fuzzy environment as it considers the element of compromise solution between the positive and negative aspect of the evaluation simultaneously. It is considered as an improvised version of Fuzzy TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method for solving decision-making problems. However, the FIR utilizes the distance approach in the evaluation of obtaining the compromise solution. A defuzzification process is carried out to transform the fuzzy values into a crisp form. Hence, loss of information may occur in the computation. In this paper, we proposed a similarity-based FIR that overcomes the above-mentioned problem. A new compromise solution for the proposed FIR is developed and an improvised procedure of FIR is suggested using the similarity measure approach. A comparative analysis between the distance based and the similarity-based FIR is carried out using a case study of preferred client selection for a loan application. The proposed method is found to be effective in solving decision-making problems as the utilization of similarity measure will sufficiently preserve the data information in the computational process of evaluation.

Keywords*: Compromise solution, Fuzzy Inferior Ratio, Fuzzy decision making, Similarity measure.*

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1. Introduction

Decision making is an essential process of obtaining a desired result based on a set of criteria under consideration from a set of alternatives. There are many techniques and approaches that have been introduced to provide a systematic way in finding the solutions. A multi criteria decision making method (MCDM) is one of the techniques that can be employed when conflicting benefits and cost criteria are included. According to Mulliner *et al.* (2016), the objective of MCDM is to deliver a ranking, classification, sorting, description, and choices of alternatives. The MCDM approaches have been successfully implemented in various applications such as in operational research, engineering system, management science and decision theory (Kumar *et al*., 2017). For instance, some specific applications such as in selection problem of a new hub airport (Janic & Reggiani, 2002), supplier selection (Nourianfar & Montazer, 2013), water resources planning (Opricovic, 2011), transportation planning (Ramani *et al.*, 2010) and network selection process in heterogeneous wireless (Obayiuwana & Falowo, 2015).

Since the introduction of fuzzy set theory by Zadeh (1965), the emergence of decision-making methods under fuzzy environment is timely, in order to cater for subjective and vague evaluation process. Instead of using crisp values to determine criteria weight and the rating of alternatives, the evaluation is now in the form of linguistic which is closer to human perception and thought. The first discussion on the decision making under fuzzy environment was discussed by Bellman & Zadeh (1970) and later, many MCDM methods have incorporated fuzziness in their techniques such as Fuzzy TOPSIS (Chen, 2000), Fuzzy VIKOR (Opricovic, 2011) and Fuzzy AHP (Bozbura *et al.,* 2007). Fuzzy TOPSIS is a well-known method that utilizes the position of positive and negative solutions in the computation. Thus, the result takes into consideration the compromise solution between the two. Nevertheless, in obtaining the rating index, they only consider the negative solution in calculating the final index of preference. To overcome this problem, Hadi-Vencheh & Mirjaberi (2014) proposed the Fuzzy Inferior Ratio (FIR) method as an improvement to the fuzzy TOPSIS method which now considers the positive and negative solution simultaneously in the calculation of the rating index. Since the best solution does not always indicate the most remote from the negative solution, the balance between the shortest distance to the positive ideal solution and the farthest distance from the negative ideal solution is desired. The method employs the distance approach in calculating difference. This has become a major setback since vital information may loss due to the simplification process of fuzzy values into crisp values.

Similarity measure (SM) between two fuzzy values is an important tool to compare similarity from various perspectives of fuzzy values such as distance, shape, size, geometrical properties, etc. Chen (1996) was the first to propose an SM based on the distance between fuzzy numbers for trapezoidal and triangular fuzzy numbers, in order to aggregate the decision makers' opinions. Lee (1999) applied SM to deal with fuzzy opinions using the distance between fuzzy numbers based on metric for group decision making and the difference between minimum and maximum universe of discourse. Chen & Chen (2003) proposed a new SM to calculate the degree between two generalized trapezoidal fuzzy numbers based on the centre of gravity (COG) points meanwhile, Yong *et al.* (2004) introduced a new SM based on radius of gyration to cater the drawback of the method of Chen & Chen (2003). Some variants of new SM have been proposed recently such as by Ahmad *et al.* (2018), Mohamad *et al.* (2019) and Wu *et al.* (2020). Some applications of SM in solving real problems are discussed. Hejazi *et al.* (2011) and Wei & Chen (2009) used SM in determining risk analysis in manufacturing. Zuo *et al.* (2013) implemented similarity measure-based method in diagnosing rotor fault and Niyigena *et al.* (2012) offered a similarity based procedure to solve supplier selection problem.

In this paper, a similarity-based FIR is proposed for solving fuzzy decision making problem. The SM used is adapted from Ahmad *et al.* (2018) that includes geometric distance, center of gravity, the Hausdorff distance, and Dice similarity index. Each component in the SM has its role to capture some specific features of the fuzzy numbers.

2. Preliminaries

A fuzzy set *A* in a universal set U is defined as an ordered pair:

$$A = \left\{ \left(x, \mu_A(x)\right) \middle| x \in A, \mu_A(x) \in [0,1] \right\} \tag{1}$$

where $\mu_A(x)$ is known as the membership degree to which *x* belongs to *A*. As opposed to crisp set where the characteristic function only gives the value 1 if the element is in the set and 0 when is not, the membership function of fuzzy set offers all values between 0 and 1 inclusively. A fuzzy number is a fuzzy set that is normal and convex; where normal refers to a fuzzy set with height 1 and convex is when the fuzzy set satisfies the membership inequality:

$$\mu_A\left(\lambda x + (1-\lambda) y\right) \geq \min\left(\mu_A(x), \mu_A(y)\right) \tag{2}$$

for (x,y) in A and $\lambda \in [0,1]$.

A generalized trapezoidal fuzzy number (GTFN) $\tilde{A} = (a_1, a_2, a_3, a_4, w)$ is a fuzzy set defined by a membership function $\mu_{\tilde{A}}(x) : R \to [0,1]$ where:

$$\mu_{\tilde{A}}(x) = \begin{cases} w\left(\dfrac{x-a_1}{a_2-a_1}\right) & , \quad a_1 \le x \le a_2 \\ w & , \quad a_2 \le x \le a_3 \\ w\left(\dfrac{x-a_4}{a_3-a_4}\right) & , \quad a_3 \le x \le a_4 \\ 0 & , \quad otherwise \end{cases} , \tag{3}$$

such that $a_1, a_2, a_3, a_4 \in R$, and $a_1 \le a_2 \le a_3 \le a_4$. When w = 1, the generalized trapezoidal fuzzy number is known as the standardized trapezoidal fuzzy number. In addition, when $a_2 = a_3$, it becomes a triangular fuzzy number and it is a singleton when $a_1 = a_2 = a_3 = a_4$. Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers. Then:

i) Addition: $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$;

ii) Multiplication: $\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$.

iii) Scalar multiplication: $k \otimes \tilde{A} = (ka_1, ka_2, ka_3, ka_4)$, $k > 0$.

3. Similarity Measure (SM)

There are several SMs that have been proposed by researchers to cater some specific problems and issues under fuzzy environment. In this paper, the SM proposed by Ahmad *et al.* (2018) is used. Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers. The SM between $\tilde{A}$ and $\tilde{B}$ is defined as:

$$\begin{aligned} S(\tilde{A},\tilde{B}) = & \left(1 - \frac{1}{4}\sum_{i=1}^{4} |a_i - b_i|\right) \times \left(1 - |\hat{x}_{\tilde{A}} - \hat{x}_{\tilde{B}}|\right)^{B(S_{\tilde{A}}, S_{\tilde{B}})} \\ & \times \left(\frac{2\left[(a_1+a_2)(b_1+b_2) + (a_3+a_4)(b_3+b_4)\right]}{(a_1+a_2)^2 + (a_3+a_4)^2 + (b_1+b_2)^2 + (b_3+b_4)^2} \right) \\ & \times \left(\frac{1}{1 + \left(max\{|a_1-b_1|, |a_2-b_2|, |a_3-b_3|, |a_4-b_4|\}\right)} \right) \end{aligned} \tag{4}$$

where $\hat{x}_{\tilde{A}}$ and $\hat{y}_{\tilde{A}}$ are the horizontal center of gravity (COG) of $\tilde{A}$ and $\tilde{B}$ calculated as:

$$\hat{x}_{\tilde{A}} = \frac{\hat{y}_{\tilde{A}}(a_2 + a_3) + (1 - \hat{y}_{\tilde{A}})(a_1 + a_4)}{2},$$

$$\hat{y}_{\tilde{A}} = \begin{cases} \dfrac{1}{6}\left(\dfrac{a_3 - a_2}{a_4 - a_1} + 2\right) & if \quad a_1 \ne a_4 \; , \\ \dfrac{1}{2} & if \quad a_1 = a_4 \; . \end{cases} \tag{5}$$

and $B\left(S_{\tilde{A}}, S_{\tilde{B}}\right)=\begin{cases}1 & if\ S_{\tilde{A}}+S_{\tilde{B}}>0\\ 0 & if\ S_{\tilde{A}}+S_{\tilde{B}}=0\end{cases}$ such that $S_{\tilde{A}}=a_4-a_1$ and $S_{\tilde{B}}=b_4-b_1$.

The above SM consists of four components, which are geometric distance, center of gravity, Hausdorff distance, and Dice similarity index. Using the Hausdorff distance, the geometry information in evaluating the mismatch between two generalized fuzzy numbers can be obtained by considering the geometry shape of membership function of the fuzzy numbers. This SM has the advantage of discriminating two similar shape fuzzy numbers with two different locations effectively (Ahmad *et al.*, 2018). The similarity measure $S\left(\tilde{A},\tilde{B}\right)$ satisfies the following properties:

(P1) Two fuzzy numbers $\tilde{A}$ and $\tilde{B}$ are identical if and only if $S\left(\tilde{A},\tilde{B}\right)=1$.

(P2) $S\left(\tilde{A},\tilde{B}\right)=S\left(\tilde{B},\tilde{A}\right)$

(P3) If $\tilde{A}=\left(a,a,a,a\right)$ and $\tilde{B}=\left(b,b,b,b\right)$ are real numbers, then:

$$S\left(\tilde{A},\tilde{B}\right)=\left(\frac{2ab}{a^2+b^2}\right)\left(\frac{1-|a-b|}{1+|a-b|}\right).$$

The proof of the properties is given in Ahmad *et al.* (2018) and the SM has been validated by using some benchmark sets of fuzzy numbers. It was found that the SM to be comparable with other SM with some additional advantages for some specific cases of comparison.

4. Fuzzy Inferior Ratio

The method of the Fuzzy Inferior Ratio was introduced by Hadi-Vencheh & Mirjaberi (2014) to overcome weaknesses of some compensatory methods of decision making where most consider only one of the ideal solutions in obtaining the rating index. It was highlighted that the single remotest value does not guarantee the alternative to be the desired one. Hence, a compromise solution was suggested between the positive ideal solution and the negative ideal solution simultaneously.

Let $A=\{A_1,\ldots,A_n\}$ be set of alternatives, $C=\{C_1,\ldots,C_m\}$ be set of criteria under consideration in the evaluation. The decision matrix of evaluation is defined as $D=\left[\tilde{a}_{ij}\right]$ where $\tilde{a}_{ij}=(a_{ij}^1,a_{ij}^2,a_{ij}^3,a_{ij}^4)$ is a trapezoidal fuzzy number. The pseudo steps of the FIR as given in Hadi-Vencheh & Mirjaberi (2014) are given as follows:

- Evaluate each alternative in *A* with respect to each criterion in *C* to obtain the decision matrix *D*.
- Normalize the decision matrix *D* to eliminate the dimension of the attribute.
- Determine the positive ideal solution (PIS) and the negative ideal solution (NIS)
- The difference between the alternative and the PIS/NIS is measured.
- The compromise solution is calculated to measure the extent to which each alternative is closest to PIS and is far away from NIS, simultaneously.
- The *inferior ratio* for both attributes of the shortest distance from PIS and farthest away from NIS is obtained.

4.1 Decision Making Procedure Using Similarity Based Fuzzy Inferior Ratio.

A decision making procedure using the FIR with the SM given earlier is proposed using the following steps. Some modification of the original FIR is necessary to accommodate the integration of the SM, in particular in evaluating the inferior ratio.

Step 1: Form a committee of K decision makers, D_k to evaluate the importance of n criteria C_j and the rating of m alternatives A_i. The linguistic terms and the corresponding trapezoidal fuzzy numbers used in the evaluation of criteria weight and ratings of alternatives is shown in Table 1 and Table 2 respectively.

Table 1: Linguistic Terms for Criteria Weights

Criteria Weight	Fuzzy Number
Very low (VL)	(0.0, 0.0, 0.0, 0.1)
Low (L)	(0.0, 0.1, 0.1, 0.3)
Medium Low (ML)	(0.1, 0.3, 0.3, 0.5)
Medium (M)	(0.3, 0.5, 0.5, 0.7)
Medium High (MH)	(0.5, 0.7, 0.7, 0.9)
High (H)	(0.7, 0.9, 0.9, 1.0)
Very High (VH)	(0.9, 1.0, 1.0, 1.0)

Table 2: Linguistic Terms for Rating of Alternatives

Rating Alternative	Fuzzy Number
Very Poor (VP)	(0, 0, 0, 1)
Poor (P)	(0, 1, 1, 3)
Medium Poor (MP)	(1, 3, 3, 5)
Fair (F)	(3, 5, 5, 7)
Medium Good (MG)	(5, 7, 7, 9)
Good (G)	(7, 9, 9, 10)
Very Good (VG)	(9, 10, 10, 10)

Let $\tilde{w}_j^k = \left(w_{j1}^k, w_{j2}^k, w_{j3}^k, w_{j4}^k\right)$ and $\tilde{x}_i^k = \left(x_{i1}^k, x_{i2}^k, x_{i3}^k, x_{i4}^k\right)$ represent the weight of criteria C_j and rating of alternatives A_i given by the k-th decision-maker, D_k, respectively, with $i = 1,2,\ldots,m$, $j = 1,2,\ldots,n$ and $k = 1,,2,\ldots,K$.

Step 2: Determine the aggregated fuzzy weight of the criteria, $\tilde{w}_j = \left(w_{j1}, w_{j2}, w_{j3}, w_{j4}\right)$ with respect to C_j where:

$$\tilde{w}_j = \frac{\left(\tilde{w}_j^1 + \tilde{w}_j^2 + \ldots + \tilde{w}_j^k\right)}{k} \tag{6}$$

Step 3: Find the aggregated fuzzy rating $\tilde{x}_{ij} = \left(\tilde{x}_{ij}^{(1)}, \tilde{x}_{ij}^{(2)}, \tilde{x}_{ij}^{(3)}, \tilde{x}_{ij}^{(4)}\right)$ of the i-th alternative, A_i with respect to j-th C_j where:

$$\tilde{x}_{ij} = \frac{\left(\tilde{x}_{ij}^1 + \tilde{x}_{ij}^2 + \ldots + \tilde{x}_{ij}^k\right)}{k} \tag{7}$$

Step 4: Construct the fuzzy decision matrix, $D = [\tilde{x}_{ij}]_{mxn}$, and the normalized fuzzy decision matrix $\tilde{R} = \left[\tilde{r}_{ij}\right]_{m\times n}$ where:

$$\tilde{v}_{ij} = \left(\frac{x_{ij}^{(1)}}{x_j^{(4)}}, \frac{x_{ij}^{(2)}}{x_j^{(4)}}, \frac{x_{ij}^{(3)}}{x_j^{(4)}}, \frac{x_{ij}^{(4)}}{x_j^{(4)}} \right), \quad x_j^{(4)} = max_{\ 1 \le i \le n}\ x_j^{(4)}\ , j \in B \tag{8}$$

$$\tilde{v}_{ij} = \left(\frac{x_j^{(1)}}{x_{ij}^{(4)}}, \frac{x_j^{(1)}}{x_{ij}^{(3)}}, \frac{x_j^{(1)}}{x_{ij}^{(2)}}, \frac{x_j^{(1)}}{x_{ij}^{(1)}} \right), \quad x_j^{(1)} = min_{\ 1 \le i \le n}\ x_j^{(1)}\ , j \in C \tag{9}$$

with *B* and *C* representing benefit and cost criteria, respectively.

Step 5: Transform the normalized fuzzy decision matrix via linear scale transformation by letting $r_{min} = 0$ and $r_{max} = 1$. Then, construct the weighted transformed normalized fuzzy decision matrix $\tilde{V}$ by multiplying the weight, $\tilde{w}_j$ of evaluation criteria with the transformed normalized fuzzy decision matrix $\tilde{r}_{ij}$ as:

$$\tilde{V} = \left[\tilde{v}_{ij}\right]_{mxn} \text{ where } \tilde{v}_{ij} = \tilde{r}_{ij} \times \tilde{w}_j = \left(v_{ij}^{(1)}, v_{ij}^{(2)}, v_{ij}^{(3)}, v_{ij}^{(4)}\right). \tag{10}$$

Step 6: Determine Fuzzy Positive Ideal Solution (FPIS), A^+ and Fuzzy Negative Ideal Solution (FNIS) A^- .

$$A^+ = \tilde{v}_j^+ = \left(max_{\ 1 \le i \le n}\ v_{ij}^{(1)}, max_{\ 1 \le i \le n}\ v_{ij}^{(2)}, max_{\ 1 \le i \le n}\ v_{ij}^{(3)}, max_{\ 1 \le i \le n}\ v_{ij}^{(4)}\ \right) \tag{11}$$

$$A^- = \tilde{v}_j^- = \left(min_{\ 1 \le i \le n}\ v_{ij}^{(1)}, min_{\ 1 \le i \le n}\ v_{ij}^{(2)}, min_{\ 1 \le i \le n}\ v_{ij}^{(3)}, min_{\ 1 \le i \le n}\ v_{ij}^{(4)}\ \right) \tag{12}$$

Step 7: Obtain the similarity values S_i^+ and S_i^- for the *i*-th alternative from FPIS, A^+ and FNIS, A^- respectively where:

$$S_i^+\left(A_i, A^+\right) = \sum_{j=1}^{n} S_i\left(\tilde{v}_{ij}, \tilde{v}_j^+\right),\ S_i^-\left(A_i, A^-\right) = \sum_{j=1}^{n} S_i\left(\tilde{v}_{ij}, \tilde{v}_j^-\right),\ i = 1, 2, ..., m\,. \tag{13}$$

Step 8: Calculate the similarity-based compromise solution for each alternative. The new compromise solution based on similarity measure, $\zeta(A_i)$ is given as:

$$\zeta(A_i) = \zeta^+(A_i) - \zeta^-(A_i)\,, i = 1, 2, ..., n \tag{14}$$

where:

$$\zeta^+(A_i) = \frac{S_i^+\left(A_i, A^+\right)}{\max_{1 \le i \le n} S_i^+\left(A_i, A^+\right)} \quad \text{and } \zeta^-(A_i) = \frac{S_i^-\left(A_i, A^-\right)}{min_{1 \le i \le n} S_i^-\left(A_i, A^-\right)}.$$

The compromise solution, $\zeta_p(A_i)$ measures simultaneously the extent to which the alternative A_i similar to FPIS and is dissimilar from FNIS. The smaller value of similar based compromise solution, $\zeta(A_i)$ such that the smaller difference of $\zeta^+(A_i)$ and $\zeta^-(A_i)$, the more preferred the alternative is. We shall show that $\zeta(A_i)$ is always non-positive.

Proposition: $\zeta(A_i) \le 0, i = 1, \ldots, n$.

Proof. Since $S_i^+\left(A_i,A^+\right)\le \max_{1\le i\le n} S_i^+\left(A_i,A^+\right)$ and $S_i^-\left(A_i,A^-\right)\ge \min_{1\le i\le n} S_i^-\left(A_i,A^-\right)$, thus we have:

$$\zeta^+\left(A_i\right)=\frac{S_i^+\left(A_i,A^+\right)}{\max_{1\le i\le n} S_i^+\left(A_i,A^+\right)}\le 1 \quad \text{and} \quad \zeta^-\left(A_i\right)=\frac{S_i^-\left(A_i,A^-\right)}{\min_{1\le i\le n} S_i^-\left(A_i,A^-\right)}\ge 1 .$$

This gives Equation (14) $\zeta\left(A_i\right)=\zeta^+\left(A_i\right)-\zeta^-\left(A_i\right)\le 1-1=0$ as required.

Step 9: Calculate the inferior ratio $IR(A_i)$ for each alternative where:

$$IR\left(A_i\right)=\frac{\zeta\left(A_i\right)}{min_{1\le i\le n}\zeta\left(A_i\right)} \tag{15}$$

The ranking order of the alternatives is determined where the least value of $IR\left(A_i\right)$ will be placed in first ranking position.

5. Implementation

As an illustration, a decision making problem of a financial institution in evaluating some loan applications is solved using the proposed procedure. A committee of three experts, DM_1, DM_2 and DM_3 evaluate six applicants (A_1, A_2, A_3, A_4, A_5 and A_6) based on four benefit criteria, which are credit history (C_1), capacity (C_2), capital (C_3) and collateral (C_4). The evaluations are made by the decision makers in determining the weight of criteria and the rating of the six applicants and are shown in Table 3 and Table 4 respectively.

Table 3: Evaluation of Importance of Criteria Weight.

	DM_1	**DM_2**	**DM_3**	**Aggregated fuzzy weight**
C_1	H	VH	VH	(0.83,0.97,0.97,1.00)
C_2	H	H	MH	(0.63,0.83,0.83,0.97)
C_3	VH	M	H	(0.63,0.80,0.80,0.90)
C_4	H	M	M	(0.43,0.63,0.63,0.80)

Table 4: Evaluation of Importance of Ratings for Six Applicants.

Applicants	**Criteria**	**DM_1**	**DM_2**	**DM_3**
A_1	C_1	F	G	F
	C_2	MP	MG	F
	C_3	P	MG	MP
	C_4	MP	MG	MP
A_2	C_1	P	F	F
	C_2	F	MG	MP
	C_3	F	MG	F
	C_4	F	MG	F

A_3	C_1	MG	MG	MG
	C_2	G	MG	MG
	C_3	MG	MG	F
	C_4	MG	MG	F
A_4	C_1	MG	MG	MG
	C_2	MG	MG	MG
	C_3	MG	MG	F
	C_4	MG	MG	F
A_5	C_1	F	MP	MG
	C_2	MG	MG	G
	C_3	F	MG	MG
	C_4	F	MG	MG
A_6	C_1	MG	F	VG
	C_2	F	F	G
	C_3	F	MG	VG
	C_4	F	MG	G

The evaluations in Table 4 are converted into fuzzy numbers, which then are aggregated into a fuzzy decision matrix using Equation (7) as shown in Table 5.

Table 5: Aggregated Fuzzy Decision Matrix.

	C_1	**C_2**	**C_3**	**C_4**
A_1	(4.33,6.33,6.33,8.00)	(3.00,5.00,5.00,7.00)	(2.00,3.67,3.67,5.67)	(2.33,4.33,4.33,6.33)
A_2	(2.00,3.67,3.67,5.67)	(3.00,5.00,5.00,7.00)	(3.67,5.67,5.67,7.67)	(3.67,5.67,5.67,7.67)
A_3	(5.00,7.00,7.00,90.0)	(5.67,7.67,7.67,9.33)	(4.33,6.33,6.33,8.33)	(4.33,6.33,6.33,8.33)
A_4	(5.00,7.00,7.00,9.00)	(5.00,7.00,7.00,9.00)	(4.33,6.33,6.33,8.33)	(4.33,6.33,6.33,8.33)
A_5	(3.00,5.00,5.00,7.00)	(5.67,7.67,7.67,9.33)	(4.33,6.33,6.33,8.33)	(4.33,6.33,6.33,8.33)
A_6	(5.67,7.33,7.33,8.67)	(4.33,6.33,6.33,8.0)	(5.67,7.33,7.33,8.67)	(5.00,7.00,7.00,8.67)

The elements in Table 5 are normalized using Equation (8) as in Table 6.

Table 6: Normalized Fuzzy Decision Matrix.

	C_1	**C_2**	**C_3**	**C_4**
A_1	(0.48,0.70,0.70,0.89)	(0.32,0.54,0.54,0.75)	(0.23,0.42,0.42,0.65)	(0.27,0.50,0.50,0.73)
A_2	(0.22,0.41,0.41,0.63)	(0.32,0.54,0.54,0.75)	(0.42,0.65,0.65,0.88)	(0.42,0.65,0.65,0.88)
A_3	(0.56,0.78,0.78,1.00)	(0.61,0.82,0.82,1.00)	(0.50,0.73,0.73,0.96)	(0.50,0.73,0.73,0.96)
A_4	(0.56,0.78,0.78,1.00)	(0.54,0.75,0.75,0.96)	(0.50,0.73,0.73,0.96)	(0.50,0.73,0.73,0.96)
A_5	(0.33,0.56,0.56,0.78)	(0.61,0.82,0.82,1.00)	(0.50,0.73,0.73,0.96)	(0.50,0.73,0.73,0.96)
A_6	(0.63,0.81,0.81,0.96)	(0.46,0.68,0.68,0.85)	(0.65,0.85,0.85,1.00)	(0.58,0.81,0.81,1.00)
w	(0.83,0.97,0.97,1.00)	(0.63,0.83,0.83,0.97)	(0.63,0.80,0.80,0.90)	(0.43,0.63,0.63,0.80)

The aggregated weighted transformed normalized values are obtained by using Equation (10) as shown in Table 7.

Table 7: Aggregated Weighted Transformed Normalized Fuzzy Decision Matrix.

	C_1	C_2	C_3	C_4
A_1	(0.28,0.60,0.60,0.86)	(0.00,0.27,0.27,0.62)	(0.00,0.20,0.20,0.50)	(0.00,0.20,0.20,0.50)
A_2	(0.00,0.23,0.23,0.52)	(0.00,0.27,0.27,0.62)	(0.16,0.44,0.44,0.77)	(0.09,0.33,0.33,0.67)
A_3	(0.36,0.69,0.69,1.0)	(0.27,0.62,0.62,0.97)	(0.22,0.52,0.52,0.85)	(0.14,0.40,0.40,0.76)
A_4	(0.36,0.69,0.69,1.0)	(0.20,0.53,0.53,0.92)	(0.22,0.52,0.52,0.85)	(0.14,0.40,0.40,0.76)
A_5	(0.12,0.41,0.41,0.71)	(0.27,0.62,0.62,0.97)	(0.22,0.52,0.52,0.85)	(0.14,0.40,0.40,0.76)
A_6	(0.44,0.74,0.74,0.95)	(0.14,0.44,0.44,0.77)	(0.35,0.64,0.64,0.90)	(0.18,0.47,0.47,0.80)

From Table 7, the values of FPIS and FNIS are determined using Equation (11) and Equation (12) where:

$$\text{FPIS} = A^{+} = (0.44, 0.74, 0.74, 1) \text{ and FNIS} = A^{-} = (0, 0.2, 0.2, 0.5).$$

The similarity values of each alternative from the FPIS and the FNIS are calculated using Equation (13). Then, the similarity values are used to determine the proposed compromise solution as in Equation (14). Using Equation (15), the inferior ratio is calculated. Finally, the ranking of the alternatives is obtained as shown in Table 8.

Table 8: The Ranking for Six Applicants Using Similarity Based FIR.

Applicants	**Criteria**	$S(\widetilde{A}_i, A^{+})$	$\sum S(\widetilde{A}_i, A^{+})$	$S(\widetilde{A}_i, A^{-})$	$\sum S(\widetilde{A}_i, A^{-})$	$\zeta_P(A_i)$	$IR(A_i)$	**Rank**
A_1	C_1	0.62	0.96	0.21	2.97	-2.56	1.00	6
	C_2	0.15		0.77				
	C_3	0.10		0.99				
	C_4	0.10		1.00				
A_2	C_1	0.11	0.87	0.93	2.69	-2.32	0.91	5
	C_2	0.15		0.77				
	C_3	0.37		0.39				
	C_4	0.23		0.59				
A_3	C_1	0.85	2.35	0.12	1.01	-0.04	0.01	2
	C_2	0.67		0.17				
	C_3	0.51		0.27				
	C_4	0.32		0.44				
A_4	C_1	0.85	2.20	0.12	1.08	-0.17	0.07	3
	C_2	0.53		0.24				
	C_3	0.51		0.27				
	C_4	0.32		0.44				
A_5	C_1	0.31	1.81	0.47	1.35	-0.60	0.23	4
	C_2	0.67		0.17				
	C_3	0.51		0.27				
	C_4	0.32		0.44				
	C_1	0.93		0.10				
	C_2	0.36		0.40				

A_6	C_3	0.74	2.44	0.16	1.02	-0.01	0.00	1
	C_4	0.41		0.35				

Table 9 shows the inferior ratio index and the ranking of six applicants using proposed similarity based FIR and existing distance based FIR.

Table 9: The Ranking for Six Applicants Using Similarity based FIR and Existing FIR.

Applicants	**Similarity based FIR**		**Existing FIR**	
	$IR_p(A_i)$	Rank	$IR_p(A_i)$	Rank
A_1	1.00	6	1.00	6
A_2	0.91	5	0.97	5
A_3	0.01	2	0.01	2
A_4	0.07	3	0.08	3
A_5	0.23	4	0.29	4
A_6	0.00	1	0.00	1

According to the FIR method, the least index value would be the most preferred alternatives. From Table 9, Applicant 6 (A_6) is the most preferred applicant and Applicant 1 (A_1)) is the least preferred applicant. It is found that the rank of the applicants is consistent for both methods and the index value is comparable to each other for each alternative. However, the similarity based FIR has an advantage over the existing distance based FIR because it minimizes the loss of information as compared to the distance based FIR as no simplification using defuzzification has been made.

6. Conclusion

The emergence of new methods in solving decision making problem systematically has helped many sectors to improve their productivity. Nevertheless, the distance approach is usually employed in obtaining rating of alternative which may dissipate some of the information inside the fuzzy values. In addition, some methods such as fuzzy TOPSIS and fuzzy VIKOR use only one remotest value to obtain the preference index of alternatives which does not guarantee the ultimate result. The FIR method was suggested by Hadi-Vencheh & Mirjaberi (2014) based on compromise solution used the both positive and negative ideal solution simultaneously. In this paper, the method of FIR is further improved by integrating the similarity measure in the decision making procedure. This is to minimize the loss of information due to simplification. A decision making procedure was developed by introducing a new similarity based compromise solution and inferior ratio. An application was illustrated to validate its effectiveness. The similarity based approach has the potential to be extended to other fuzzy decision making methods that utilise distance method in the computation.

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